Equity beta for a benchmark efficient water utility

Report prepared for Seqwater | 17 June 2021

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1 Executive Summary

1.1 Author of report

1. This report has been authored by Professor Stephen Gray, Professor of Finance at the UQ Business School, University of Queensland and Director of Frontier Economics, a specialist economics and corporate finance consultancy. I have Honours degrees in Commerce and Law from the University of Queensland and a PhD in Financial Economics from Stanford University. I teach a range of courses in finance, corporate valuation and econometrics. I have published widely in high-level academic journals, and I have more than 20 years of practical finance consulting experience.

2. My teaching, research and consulting experience extends to issues relating to statistical analysis and econometric modelling. I have published widely in the areas of financial econometrics and empirical finance, including papers in relation to the estimation of WACC parameters. I have also prepared numerus reports for firms and regulatory bodies relating to the estimation of regulatory WACC parameters.

3. A copy of my curriculum vitae has been provided with this report.

4. My opinions set out in this report are based on the specialist knowledge acquired from my training and experience set out above. I have been provided with a copy of Chapter 11 Part 5 of the Queensland Uniform Civil Procedure Rules 1999. I have read, understood and complied with the Rules.

5. I have been assisted in the preparation of this report by Dinesh Kumareswaran and James Key from Frontier Economics.

1.2 Summary of conclusions

6. In previous determinations for Seqwater, the QCA has adopted equity beta estimates generated by Incenta. Incenta's approach is to identify a set of comparator businesses, use regression analysis to estimate the equity beta for each firm in the set, re-lever to the assumed 60% gearing, and take the mean of the resulting estimates as its point estimate. Incenta also considers a second set of comparator firms and adopts the mean re-levered equity beta from that set as its upper bound estimate.

7. I have updated the point and upper bound estimates, following the Incenta approach, to reflect the most recently available data. I find that neither the point estimate nor the upper bound have changed materially since the QCA's last review. The point estimate is 3 basis points lower and the upper bound is 12 basis points higher. Both changes are statistically insignificant – being tiny relative to the degree of statistical estimation error involved in this process. That is, these changes in estimates are highly likely due to random statistical estimation error, rather than a real shift in the true systematic risk of the comparator firms.

8. I note that a clear outcome from IPART's recent review of its beta estimation approach is the recognition that stakeholders highly value regulatory certainty and predictability over time.

Consequently, IPART determined that it would change its beta allowance only when there is sufficient evidence to do so:

> *We acknowledge stakeholders' concerns that the equity beta should only be changed in response to significant evidence, in order to maintain certainty. Therefore we have decided to review the equity beta at each price review (currently every 4-5 years), but only change it when there is sufficient evidence that our existing estimate is no longer appropriate.*[1]

9. This is a sensible approach, in my view. The updated Incenta estimates certainly would fall well short of 'sufficient evidence' to support changing the allowed equity beta.

10. I also identify a number of potential improvements to the approach currently adopted by Incenta – designed to improve the robustness of equity beta estimates and to reduce the impact of random statistical estimation error. These include:

    a **Consideration of other reference days.** Incenta's beta estimates are derived using weekly and monthly returns data. These returns are computed using a single reference day (e.g., weekly returns are only based on Friday to Friday observations). Returns data using all other available reference days (e.g. Monday to Monday, Tuesday to Tuesday, and so on) are ignored in the estimation. This introduces significant sampling error and random noise in the final beta estimates. This, in turn, reduces the reliability of Incenta's beta estimates considerably.

    b **Use total debt to compute gearing of comparators.** Incenta computes the gearing of the comparators (for the purposes of de-levering betas) using net debt rather than total debt. In doing so, Incenta implicitly makes the extreme and unrealistic assumption that the comparators can use all available cash and cash equivalents to retire debt. No efficient and prudent business would adopt such an approach as doing so would leave the business with insufficient working capital to meet its day-to-day operating requirements.

    c **De-levering and re-levering equations.** Incenta uses de-levering and re-levering formulas that include a tax rate term. These formulas are incompatible with the QCA's assumption that the benchmark efficient water utility will maintain a constant gearing level. Under such an assumption, the appropriate de-levering and re-levering formulas should omit the tax rate term altogether.

    d **Inclusion of illiquidity and data quality filters.** When selecting its comparator sample, Incenta does not apply any filters to remove stocks that are illiquid or firms that have insufficient historical data to produce reliable beta estimates. The inclusion of illiquid stocks will tend to bias beta estimates down, while the inclusion of stocks with insufficient data will add statistical noise to the estimates.

    e **Adjustments to raw betas to correct statistical bias.** Incenta makes now adjustments to raw beta estimates obtained from Bloomberg. However, given the statistical imprecision of beta estimates, it is standard practice to adjust beta estimates towards a prior value. I favour the Vasicek adjustment, which corrects raw beta estimates in inverse proportion to the statistical precision of the raw estimates.

[1] IPART, Review of our WACC method – Final report, February 2018, p. 61.

- f **Expanded comparator sample.** Incenta relies on just 12 water company comparators to derive its equity beta point estimate. Expanding the comparator set to include other relevant comparators is likely to improve the reliability and stability of beta estimates.

11. I show that the adoption of these improvements to the beta estimation process currently have the effect of increasing the equity beta estimates for water businesses above 0.77.

12. In summary, I note that the evidence does not support a reduction in the equity beta, but rather that the most robust evidence suggests that, if any change is to be made, the allowed equity beta should be increased. In my view, the evidence currently supports a re-levered equity beta of at least 0.9.

2 Updated estimates using Incenta approach

13. This section updates the estimates produced by Incenta for the QCA, for the purposes of the Seqwater Bulk Water Price Review 2018-21 decision.

2.1 Approach

14. The QCA engaged Incenta to provide advice on an appropriate estimate of the equity beta for the Seqwater Bulk Water Price Review 2018-21.[2] Incenta obtained its equity beta estimates for each comparator directly from Bloomberg, rather than performing its own estimation using raw data on each comparator. The methodology underpinning Incenta's estimates is summarised below in **Table 1**.

[2] Estimating Seqwater's firm-specific WACC parameters for the 2018-21 bulk water price investigation, November 2017 (Incenta Report).

Table 1: Beta estimation approach followed by Incenta

Issue	Incenta/QCA approach
Minimum number of months of data required	No formal rule specifying minimum number of months of data required
Liquidity filter	No formal rule to filter out illiquid stocks
Comparators	12 water industry comparators located in the US and UK (point estimate) 5 toll road comparators (upper bound)
Estimation period	Use longest history available over a 10-year estimation period
Returns frequency	Monthly and weekly returns
Reference days	A single reference day for each returns frequency
De-levering/re-levering formulas	Use formulas that account for corporate tax rates: Average effective tax rates over 15-year period for overseas comparators; statutory tax rate, adjusting for gamma, for Australian comparators.
Adjustments to raw betas	No Blume or Vasicek adjustments
Gearing for de-levering betas	For each comparator gearing calculated using net debt
Averaging period for gearing	Length of estimation period

Source: Incenta, Estimating Seqwater's firm-specific WACC parameters for the 2018-21 bulk water price investigation, November 2017.

15. Applying the approach outlined above, Incenta advised that the best empirical estimate of the asset beta was 0.41, with an upper bound of 0.47. The best estimate was based on 12 water utility comparators firms listed in the United States and United Kingdom. The upper bound estimate was based on five toll road comparators drawn from various countries, including Australia. The estimation period was the 10 years to June 2017.

16. In this report I have updated Incenta's estimates by:

    a Adopting the gamma and debt beta parameters used by the QCA in its most recent decisions;

    b Shifting the estimation period to the 10 years to June 2019; and c Expanding the toll road comparator as Incenta did in a recent report to the QCA.[3]

17. As noted above, in its previous reports for the QCA, Incenta obtained raw equity betas directly from Bloomberg (rather than performing its own estimation) and applied no adjustments to those raw beta estimates. For the purposes of updating Incenta's estimates, I followed the same approach.

18. Raw equity beta estimates derived using weekly and monthly returns were obtained.4

19. Incenta provided beta estimates for the Queensland Rail's 2020 Draft Access Undertaking (QRL 2020 DAU).5 That report used 12 water comparators and 7 toll road comparators.6 I assume that Incenta would use the same comparator group as it did in the QRL 2020 DAU, if it were to update its estimates today. Thus, for the purposes of presenting updated estimates using the Incenta approach I adopt the same comparator set.

20. To de-lever the raw betas I follow Incenta's approach of:

a using average annual net debt and market capitalisation as at the end of each calendar year, for each comparator;[7]

b using an estimate of long-term effective tax rates obtained from Bloomberg, for any overseas comparators;[8]

c applying the Conine equation;[9]

d applying the statutory tax rate, adjusted using Incenta's estimate of gamma, for any Australian comparators. Incenta's latest published estimate of gamma (applied by the QCA in the draft QRL 2020 DAU decision) is 0.484;[10]

e applying Incenta's latest published estimate of the debt beta. In the draft QRL 2020 DAU decision, Incenta recommended a debt beta estimate of 0.12, which was accepted by the QCA.[11]

21. In obtaining the updated equity beta estimate I retained the same benchmark gearing applied by QCA in the 2018-21 review to re-lever betas, 60%. I note that a recent Frontier Economics report

[3] Incenta, Estimating Queensland Rail's WACC for the 2020 DAU – asset beta, benchmark gearing, and credit rating, April 2019 (Incenta 2020 DAU).

[4] For the purposes of this report we did not obtain rolling 5-year beta estimates.

[5] Incenta 2020 DAU.

[6] At our recommendation, Incenta added two toll roads comparators to the sample it used when estimating Seqwater's beta: Atlas Arteria Roads and Groupe Eurotunnel SE. See Incenta 2020 DAU, p. 15.

[7] Using net debt and market capitalization, obtained from Bloomberg.

[8] The average of effective tax rates over the period 1995-2009.

[9] The Conine equation specifies the following relationship between the equity, asset and debt betas: $\beta_e = \beta_a\left(1+(1-T)\frac{D}{E}\right) - \beta_d(1-T)\frac{D}{E}$.

[10] Incenta 2020 DAU, p. 18.

[11] QRL 2020 DAU, p. 29.

prepared for Seqwater demonstrates that the evidence continues to support the use of 60% gearing for Seqwater.[12]

2.2 Results

22. The updated estimates using Incenta's approach for the water comparators are provided below in **Table 2**, and the updated estimates for the toll road comparators are presented in **Table 3**.

Table 2: Water - individual firm asset betas

Name	Ticker	Country	Weekly	Monthly
American States Water Co	AWR US Equity	US	0.41	0.15
American Water Works Co Inc	AWK US Equity	US	0.54	0.17
Artesian Resources Corp	ARTNA US Equity	US	0.39	0.11
California Water Service Group	CWT US Equity	US	0.43	0.25
Connecticut Water Service Inc	CTWS US Equity	US	0.24	0.12
Essential Utilities Inc	WTRG US Equity	US	0.62	0.41
Middlesex Water Co	MSEX US Equity	US	0.57	0.33
Pennon Group PLC	PNN LN Equity	UK	0.46	0.31
SJW Group	SJW US Equity	US	0.60	0.32
Severn Trent PLC	SVT LN Equity	UK	0.40	0.34
United Utilities Group PLC	UU/ LN Equity	UK	0.37	0.28
York Water Co/The	YORW US Equity	US	0.68	0.31
Mean			**0.48**	**0.26**
Median			**0.45**	**0.29**

Source: Bloomberg and Frontier Economics analysis.

[12] Frontier Economics, Gearing for a benchmark efficient water utility, April 2021.

Table 3: Toll roads - individual firm asset betas

Name	Ticker	Country	Weekly	Monthly
ASTM SpA	AT IM Equity	Italy	0.53	0.71
Abertis Infraestructuras SA	ABE SM Equity	Spain	0.47	0.45
Atlantia SpA	ATL IM Equity	Italy	0.60	0.63
Atlas Arteria Ltd	ALX AU Equity	Australia	0.70	0.83
Getlink SE	GET FP Equity	France	0.54	0.56
Societa Iniziative Autostradal	SIS IM Equity	Italy	0.51	0.63
Transurban Group	TCL AU Equity	Australia	0.48	0.41
Mean			**0.55**	**0.60**
Median			**0.53**	**0.63**

Source: Bloomberg and Frontier Economics analysis.

23. In past reports to the QCA, Incenta has derived what it considers to be the 'best' point estimate of beta taking an equal-weighted average of the mean weekly and mean monthly estimates for the water comparator sample.[13] That approach currently produces an estimate of 0.37. This represents a slight reduction from the estimate Incenta produced for the Seqwater Bulk Water Price Review 2018-21 decision, 0.41.

24. Re-levering the updated water comparator asset beta estimate using the Conine equation, applying QCA gamma and debt beta estimates, 60% benchmark gearing and statutory tax of 30% yields an updated equity beta point estimate of 0.68. This is somewhat lower than the equity beta estimate the QCA adopted in the Seqwater Bulk Water Price Review 2018-21 decision, 0.77.[14]

25. In past work for the QCA, Incenta has derived the 'upper bound' estimate by taking an equal-weighted average of the mean weekly and mean monthly estimates for the toll road comparator sample. Following this approach, currently produces an estimate of 0.58. This is somewhat higher than the upper bound estimate derived by Incenta for the Seqwater Bulk Water Price Review 2018-21 decision, 0.47. The increase is partially driven by the expanded sample of toll road comparators, since the five firms used in the Seqwater Bulk Water Price Review 2018-21 decision would yield an estimate of 0.54 compared to the value of 0.47 in the Incenta report. Re-levering the upper bound asset beta estimate of 0.58 yields an upper equity beta estimate of 1.15, compared to 0.91 previously.[15]

[13] Incenta Report, pp. 26-27.

[14] The higher gamma estimate, 0.484 vs 0.46, had minimal impact on beta estimates, having the effect of increasing re-levered equity beta by 0.03.

[15] Incenta Report, p. 30.

2.3 Implications

26. In the draft Seqwater Bulk Water Price Review 2018-21 decision, the QCA concluded that the best estimate of the equity premium was 5.39%, equal to the MRP estimate multiplied by Incenta's best re-levered beta estimate of 0.77.[16] In its final decision the QCA determined an equity premium of 5.36% after updating the MRP to reflect the QCA's revised best estimate.[17] That is, the QCA appears to have adopted what Incenta considered to be the best equity beta estimate, based on water comparators only, to determine the equity premium for Seqwater.

27. My update of Incenta's estimates for the water company comparator sample suggests a small reduction in the beta estimate from 0.77 to 0.68. However, there is so much statistical noise around these beta estimates that the QCA should have little confidence that a change in the point estimate as small as 0.09, based on a sample of just 12 water comparators firms, represents a genuine change in the true beta of a benchmark efficient water company.

28. For context, Figure 1 below plots the updated re-levered equity betas obtained for the 12 water comparators from Bloomberg, with standard statistical confidence intervals set as two standard errors above and below the point estimate.[18] The average, 0.68, is included for comparison. The wide range of the confidence intervals, particularly those using monthly returns (a consequence of having a quarter of the observations of samples using weekly returns), highlights the imprecision associated with beta estimation, and the insignificance of difference of only 0.09 between two sets of estimates.[19]

29. Moreover, I note that the mean of the (more precise) weekly estimates is greater than 0.68 and that it is the (less precise) monthly estimates that reduces the overall mean.

30. It is also possible to derive the standard error of the mean estimate by assuming that the twenty-four individual re-levered equity beta estimates, as shown in Figure 1, are normally distributed and independent.20 Under these assumptions, the width of the standard confidence interval is 0.14. This further highlights the insignificance of the difference of 0.09 between the most recent Incenta figures and my updated analysis.

[16] QCA, Seqwater Bulk Water Price Review 2018–21 draft report, November 2017, p. 56.

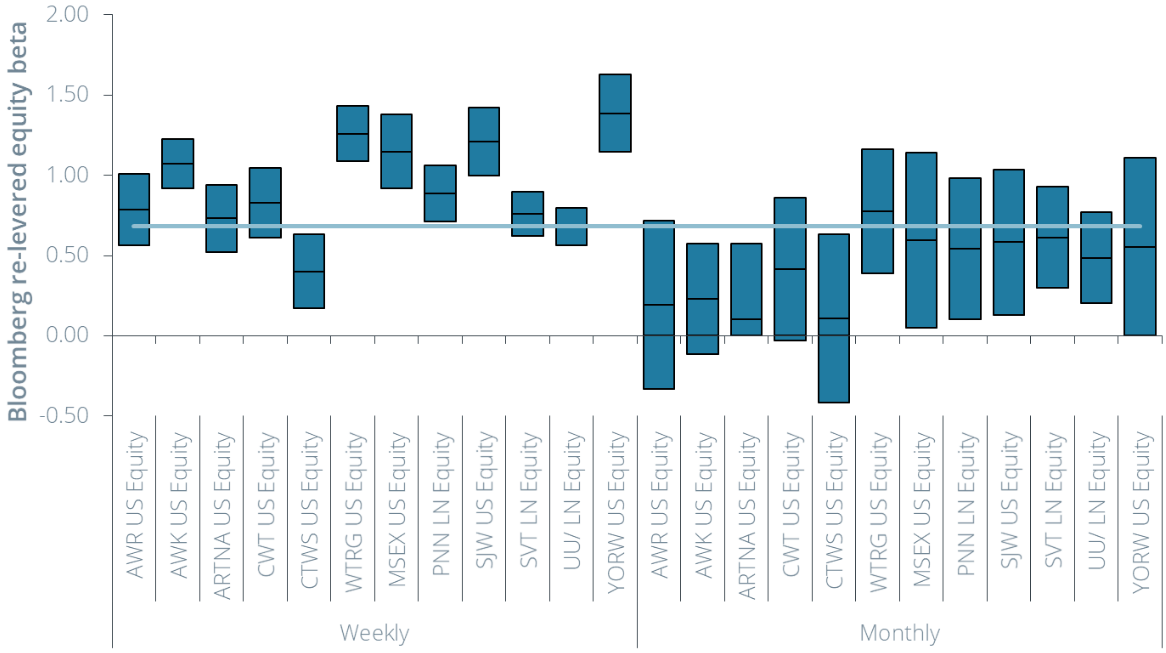
[17] QCA, Seqwater Bulk Water Price Review 2018–21 final report, March 2018, p. 62.

[18] These raw equity beta estimates should not be directly compared to 0.77 or 0.68 due to differences in gearing and tax structure.

[19] The imprecision associated with the point estimate of 0.68 should also reflect the sensitivity of the estimate to reference days, as discussed in Section 4.1.

[20] The independence assumption is likely to be violated: estimates are plausibly positively correlated, which would increase the standard error of the mean equity beta estimate.

Figure 1: **Confidence intervals of individual re-levered Bloomberg equity beta estimates**



Source: Bloomberg

2.4 Stability over time

31. In its 2018 WACC methodology decision, IPART recognised that stakeholders' value regulatory certainty and predictability over time and therefore decided that, whilst it would review its beta estimates from time to time, it would change its estimate only when there is sufficient evidence to do so:

> *We acknowledge stakeholders' concerns that the equity beta should only be changed in response to significant evidence, in order to maintain certainty. Therefore we have decided to review the equity beta at each price review (currently every 4-5 years), but only change it when there is sufficient evidence that our existing estimate is no longer appropriate.*[21]

32. This is a sensible approach, in my view. The *true* level of systematic risk exposure of regulated businesses is likely to be fairly stable over time (since the fundamental risk characteristics of firms, particularly the types of firms regulated by the QCA, generally change slowly). However, it is not possible to *observe* the true level of systematic risk—it is only possible to *estimate* it, typically with material statistical imprecision, as shown above. Therefore, large changes in beta *estimates* over time are more likely to be due to statistical noise and sampling error than genuine changes in systematic risk.

33. Consequently, I recommend that the QCA apply updated beta estimates to determine the rate of return allowance in a manner that is likely to minimise, rather than amplify, statistical noise in beta estimates. That is, the equity beta parameter used in calculating the appropriate rate of

[21] IPART, Review of our WACC method – Final report, February 2018, p. 61.

return allowance should only increase or decrease if there is sufficient evidence to warrant a change from the beta used in previous decision. Such an approach will tend to produce relatively stable estimates over time that more likely mirrors the true pattern of systematic risk exposure of regulated businesses.

34. In the remainder of this report, I make a number of recommendations aimed at improving the statistical reliability and stability of beta estimates over time.

3 Potential improvements to the Incenta approach

35. This section discusses a number of important modelling choices that arise when estimating beta—namely, the length of the estimation period, the frequency of returns data, and the way in which returns are measured. I also identify three issues related to Incenta's approach to de-levering and re-levering betas and explain how those issues might be addressed. I also discuss how the Incenta sample of water company comparators could be expanded to obtain more reliable and stable estimates over time.

3.1 Estimation period

36. I note that Incenta applied a 10-year estimation period for beta estimation on the basis that shorter estimation periods would be likely to provide inferior estimates of the forward-looking asset beta.[22] I agree with the approach of using a relatively long estimation period. Such an approach has two key benefits:

   a The resulting estimates for individual comparators are likely to be more statistically precise, since long estimation windows permit the inclusion of more data points. For instance, a 10-year period would permit the use of up to 120 monthly historical returns for a given stock. By contrast, a five-year period would permit just 60 monthly historical returns; and

   b As discussed in section 2.4, the true level of systematic risk for individual stocks tends to be fairly stable over time. Using relatively long estimation periods will tend to produce more stable estimates over time. Beta represents the average relationship between stock returns and market returns. The longer the estimation period, the greater the horizon over which this average relationship will be estimated, and the less variable the estimated relationship will be when estimates are measured periodically.

37. However, it is my view that a longer estimation period than has been employed by Incenta would further improve the stability of estimates. As such, I would recommend an estimation period of at least 15 years to promote stability in estimates over time and to provide more regulatory certainty. For the purposes of compiling the estimates set out below, I applied a 20-year estimation window. A number of comparators had historical information that spanned significantly fewer than 20 years. In such cases, estimates were derived using all the historical data available within the 20-year estimation period, provided that a minimum of 60 months of data are available.

[22] Incenta Report, p. 24.

3.2 Frequency of returns

38. I agree with Incenta that weight should be not be placed on estimates obtained using daily windows.23 A well-recognised problem with daily returns data is that the observed returns can be correlated over time due to 'non-trading.' It is well recognised in the finance literature that this has the effect of creating a statistical bias in beta estimates.24

39. While Incenta applies equal weighting to weekly and monthly return windows, I recommend using weekly returns due to the increased statistical precision afforded by the larger sample size. In this regard, I note that the AER now considers that the most useful empirical estimates use weekly return intervals.25

40. However, if the QCA is minded to retain the use of monthly returns data, I agree that it would be appropriate to obtain an overall estimate for each comparator by averaging the monthly and weekly estimates, per Incenta's current approach.

3.3 Reference days

41. When estimating betas using weekly or monthly returns data, it is necessary to choose the 'reference day' used to calculate returns (e.g., in the case of weekly data, Monday-to-Monday, Tuesday-to-Tuesday, etc.).

42. The resulting beta estimates can be highly sensitive to the reference days selected. The risk of estimation error due to the choice of reference day is known in the empirical finance literature as reference day risk. Acker and Duck (2007), who investigated the extent of reference day risk associated with five-year monthly betas for S&P500 companies using Datastream data, show that the effect of reference day risk can be severe.26 For example, they found that:

    a The estimated beta of one stock was +2 using one reference day and -2 using another;

    b Between two consecutive five-year periods, the estimated beta of one stock fell by 0.93 using one reference day and rose by 3.5 using another; and

    c The average difference in the beta estimate (arising from a change in the reference day used to measure returns), across all stocks in the sample, ranged between 0.70 and 0.92, depending on the five-year estimation window considered.

43. Dimitrov and Govindaraj (2007) confirm the findings of Acker and Duck using a different dataset (i.e., CRSP).[27] They found, for instance, that one stock in the sample had a monthly beta estimate of 0.38 using one reference day and 2.45 using another (a difference of +2.08), over the same estimation period. In that study, the mean difference in estimated betas (across all stocks),

[23] Incenta Report, p. 23.

[24] Scholes, M. and J. Williams, 1977, Estimating betas from nonsynchronous data, *Journal of Financial Economics*, 5, 3, 309-327.

[25] AER, 2018 Rate of Return Guideline Final Decision – Explanatory Statement, Dec 2018, p. 181.

[26] Acker, D., and N. W. Duck, 2007. "Reference-day risk and the use of monthly returns data," *Journal of Accounting, Auditing and Finance*, 22, 527–557.

[27] Dimitrov, V., and S. Govindaraj, 2007. "Reference-day risk: Observations and extensions," *Journal of Accounting, Auditing and Finance*, 22, 559–572.

arising from a change in the reference day used to measure monthly returns, was +0.68, which Dimitrov and Govindaraj note is similar to the mean range found by Acker and Duck (i.e., 0.70 to 0.92).

44. Accordingly, my view is that using estimates based single reference days, as the Bloomberg raw beta estimates are, leads to unnecessary variability of beta estimates and instability over time.

45. A standard way to mitigate this sampling error problem is to estimate each comparator's beta as the mean of estimates derived using all possible reference days. For instance, if using weekly returns, then the estimation process for a given comparator would involve:

   a Deriving an estimate of beta five different reference days (Monday, Tuesday, ..., Friday); and

   b Averaging over all five estimates to obtain a mean estimate for that comparator.

46. If monthly returns data are used, then the estimation process would be repeated using every trading day within the month as the relevant reference day.

47. This exercise would then be repeated for each comparator. Such a process would utilise all the information contained in the returns data for each comparator and minimise sampling error that would introduce spurious variability in estimates over time.

3.4 De-levering and re-levering approach

48. I note that the Incenta approach to de-levering and involves the following:

   a Gearing for each comparator is obtained using net debt rather than total debt; and

   b The Conine de-levering and re-levering formulas are applied, which contain a tax rate term (adjusted for the value of imputation tax credits). However, the QCA implicitly assumes that the gearing of the benchmark efficient entity is maintained at a constant level. Under these circumstances, corporate finance theory and best practice require that the de-levering and re-levering formulas should omit the tax rate term.

49. I discuss each of these issues in turn below and recommend that the re-levering process should be revised.

3.4.1 Use of net debt to compute gearing

50. The implicit assumption when using net debt to compute gearing for each comparator is that the business could use all of its cash and cash equivalents to retire debt. Therefore, any quantities of debt that can be repaid using cash within the business should be disregarded when computing the gearing of the firm.

51. I consider that such an assumption is extreme. Businesses in the real world maintain cash balances, as part of their working capital, to carry on their day-to-day operations—including paying their operating expenses. It would be unrealistic to assume that it would be efficient for the comparator firms to use cash that is required to support operations to retire their debt.

52. For this reason, I recommend the use of total debt rather than net debt when computing the gearing of the comparator firms.28

3.4.2 De-levering and re-levering formulas

53. I note that Incenta applies the Conine equation to de-lever and re-lever beta estimates. However, these de-levering and re-levering formulas, incorporating tax terms, are appropriate only if one assumes that the firm in question maintains a constant dollar amount of debt. This is not the assumption that the QCA makes. Rather, the QCA assumes that the benchmark entity maintains a constant gearing ratio (that is, debt as a proportion of total capital).

54. Appendix A presents a mathematical proof that demonstrates that under the QCA's assumed debt financing approach, the appropriate de-levering and re-levering formulas omit the tax term. The appropriate de-levering formula under an assumption of constant gearing is:

$$\beta_a = \frac{\beta_e + \beta_d \frac{D}{E}}{1 + \frac{D}{E}}.$$

55. The analogous re-levering formula is:

$$\beta_e = \beta_a + (\beta_a - \beta_d)\frac{D}{E}.$$

56. I apply these formulae in obtaining beta estimates, applying a debt beta of 0.12 and gearing of 60%.

3.5 Liquidity and data quality rules

57. In its 2018 WACC review IPART decided to exclude thinly-traded stocks when estimating equity betas. In a subsequent 2019 consultation on its approach to estimating beta, IPART applied a number of liquidity filters:[29]

   a Remove months with less than 10 days of trading data for a given stock;

   b Exclude firm-months which exceed an Amihud threshold of 25;[30] and

   c Exclude firms with less than 36 months of valid data.

58. I consider that the above filters would serve to improve the reliability of Incenta's beta estimates. However, in my view even three years of data are insufficient to derive reliable beta estimates. In my experience, finance practitioners typically use at least five years of historical data to derive beta estimates. I therefore recommend a more stringent filtering rule by excluding any companies that have fewer than 60 months of historical data after applying liquidity filters.

59. Applying this more stringent rule would have two benefits:

[28] Further, we apply the average over observations used in estimating beta as some months may be omitted due to liquidity filters.

[29] IPART, Estimating equity beta, April 2019.

[30] IPART, Review of our WACC method, Final Report –Research, February 2018, p 62.

a Increasing the minimum data requirement would reduce measurement error and improve the reliability of estimates; and

b The median beta estimate for the sample is likely to be more stable (in part due to the reduced scope for measurement error).

3.6 Vasicek adjustment

60. In the CAPM, the average firm has, by construction, a beta of 1. A beta estimate less than 1 may reflect a true beta less than one, or simply reflect estimation error. Adjustments such as the Vasicek adjustment seek to correct for possible estimation error. The resulting estimates have been shown to have superior empirical performance to raw OLS beta estimates.[31] Further, a number of commercial data services provide default beta estimates that employ such a statistical correction.[32]

61. While Incenta did not apply a Vasicek or Blume adjustment, and the AER 2018 Guideline does not apply an adjustment, that adjustment is applied by IPART as of the 2018 WACC review, noting that:

> *Several studies have found equity betas obtained from ordinary least squares (OLS) regression analysis are likely to be subject to a high degree of estimation bias due to sampling error. Regulators commonly adjust for this bias using the Vasicek and/or Blume methods.*[33]

62. Accordingly, I apply the Vasicek adjustment.

3.7 Sample selection

63. One of the limitations of the Incenta approach is that it relies on a relatively small sample of just 12 water comparator firms. There are likely to be a number of benefits to increasing the sample of relevant comparators, including:

a Reduced impact of statistical noise, such that the sample median beta may be closer to the true beta;

b More stable estimates over time due to a larger number of observations, which would reduce statistical noise; and

c More stable estimates over time due to less sensitivity to individual firms being added or removed from sample (e.g., due to listings, de-listings or takeovers/mergers).

64. I therefore considered the feasibility of expanding the water company comparator sample.

65. My starting point for comparator selection is the water industry comparator set considered by Incenta, as shown in **Table 2**.

[31] Brooks, Diamond, Gray and Hall, The Vasicek adjustment to beta estimates in the Capital Asset Pricing Model, June 2013.

[32] Brooks, Diamond, Gray and Hall, The Vasicek adjustment to beta estimates in the Capital Asset Pricing Model, June 2013, p. 5.

[33] IPART, Review of our WACC method, February 2018, p. 7.

66. I then considered whether it would be possible to add any further comparators identified as belonging to the water industry by at least one at least one of a range of industry classification schemes. In particular, I consider the Thomson Reuters Business Classification (TRBC), the Bloomberg Industry Classification Standard (BICS), Global Industry Classification Standard (GICS), Standard Industry Classification Standard (SICS) and FTSE Industry Classification Benchmark (ICB).34

67. I note that IPART recently used the Thomson Reuters industry classification system to identify a final sample of 35 relevant water companies for the purposes of beta estimation—a sample that is materially larger than the one used by Incenta.35

68. I recognise that expanding the sample set could lower the reliability of beta estimates if the firms added to the sample are inferior comparators and therefore are less informative than the original firms in the sample. Therefore, it is important to follow a careful and systematic process for expanding the comparator set.

69. In selecting which firms to use in the final estimation, the first step is to take each firm's description from the Bloomberg website. After analysing the description of comparators provided by Bloomberg, each firm was categorised as either: 'Pure-play', 'Multi-utility' or 'No water'. A 'pure-play' firm meant that, based on the description of the firm provided by Bloomberg, it could be reasonably concluded that the firm's entire activities revolved around water or their activities were similar to those of Seqwater.

70. In practice, I first applied the liquidity filters described above, albeit with a less restrictive 10-month requirement, to expedite the exercise.

71. My candidate sample consists of 42 firms, 8 of which are Chinese. I omit the Chinese comparators, following the approach set out in the IPART fact sheet. Excluding these firms results in lower beta estimates.36

72. After applying the liquidity filters as above, requiring 60 months of valid data after removing months on the basis of Amihud or trading days, I obtain a final sample consisting of 21 firms categorised as 'pure-play' water businesses. This sample is more than double the size of the Incenta comparator set.

73. I note that Incenta restricted the water comparator sample to "English speaking countries that are likely to have closer market and institutional features." Along those lines, I consider an appropriate sensitivity to be restricting the sample to developed countries only. Accordingly, as a sensitivity analysis, I investigate what the beta estimates would be under my improved methodology if the comparator sample were constituted only of firms drawn from markets in developed countries. For this purpose, I use the Morgan Stanley Capital International (MSCI) country classification system to identify developed economies.[37] My sample of comparators drawn from developed economies comprises 15 firms in total.

[34] IPART recently used the Thomson Reuters industry classification system to identify a sample of 35 relevant water companies for the purposes of beta estimation.

[35] IPART, Estimating equity beta, April 2019, p. 7.

[36] Estimates of equity beta for the full sample are slightly higher than the sample with Chinese comparators excluded.

[37] See https://www.msci.com/market-classification.

3.8 Data source

74. The data required to estimate beta was collected from Bloomberg. For each firm, daily return indices were collected for a 20-year period from 1 June 1999 to 30 June 2019, in addition to total debt, market capitalisation, and trading volume. Daily market return indices were obtained; the relevant market index for each country is listed in **Table 4** below.[38] A 20-year period of data was collected as my recommended estimation period is the longest history available over 20 years.

Table 4: Market indices used

Country	Market Index
Brazil	IBOV Index
Chile	IPSA Index
France	CAC Index
Greece	FTASE Index
Hong Kong	HSI Index
India	SENSEX Index
Malaysia	FBMKLCI Index
Philippines	PCOMP Index
Thailand	SET Index
United Kingdom	UKX Index
United States	SPX Index
Vietnam	VNINDEX Index

Source: Bloomberg.

3.9 Comparison of approaches

75. **Table 5** below summarises the differences between the Incenta approach and the alternative approach that I recommend.

[38] For Chinese comparators we used the combined SZASHR Index rather than the Shanghai or Shenzhen specific indices.

Table 5: Comparison of approaches

Issue	Incenta/QCA approach	Frontier Economics recommendation
Minimum number of months of data required	No formal rule specifying minimum number of months of data	At least 60 months of valid historical data must be available
Liquidity filter	No formal rule to filter out illiquid stocks	Exclude firm-months identified as illiquid using an Amihud and trading days filter
Comparators	12 water industry comparators located in the US and UK	Expanded sample of 26 comparators (identified using several data service providers) that pass data/liquidity filters
Estimation period	Use longest history available over a 10-year estimation period	Use longest history available over a 20-year estimation period
Returns frequency	Estimates based on average of monthly and weekly	Estimates based on weekly and monthly returns – but use all available reference days
Reference days	A single reference day for each returns frequency	Average over estimates derived using all possible reference days
De-levering/re-levering formulas	Use formulas that account for corporate tax rates: Average effective tax rates over 15-year period for overseas comparators; statutory tax rate, adjusting for gamma, for Australian comparators.	Use de-levering and re-levering formulas that omit the tax rate terms.
Adjustments to raw betas	No Blume or Vasicek adjustment	Vasicek adjustment applied
Gearing for de-levering	Net debt, market capitalisation	Total debt, market capitalisation
Average period for gearing	Length of estimation period	Length of estimation period

Source: Incenta, Frontier Economics.

4 Beta estimates using my recommended approach

76. This section presents results of beta estimation applying my recommended approach described in section 3.

4.1 Incenta sample

77. I initially apply my recommended approach using the 12 comparators identified by Incenta as comparators in the water industry.

78. Table 6 below presents the average beta estimates for each comparator over the 20 reference days considered for the monthly frequency. Individual reference day estimates are not presented, but the standard deviation over reference days is included.

Table 6: Re-levered equity betas using monthly data

Name	Ticker	Mean re-levered equity beta	Standard deviation over reference days
AMERICAN STS.WATER	AWR US Equity	0.88	0.31
AMERICAN WATER WORKS	AWK US Equity	0.69	0.19
ARTESIAN RES.'A'	ARTNA US Equity	0.95	0.22
CAL.WATER SER.	CWT US Equity	0.86	0.23
CONNECTICUT WATER	CTWS US Equity	0.60	0.26
ESSENTIAL UTILITIES	WTRG US Equity	1.21	0.20
MIDDLESEX WATER	MSEX US Equity	1.05	0.14
PENNON GROUP	PNN LN Equity	0.71	0.07
SEVERN TRENT	SVT LN Equity	0.71	0.09
SJW GROUP	SJW US Equity	1.09	0.13
UNITED UTILITIES GROUP	UU/ LN Equity	0.71	0.10
YORK WATER	YORW US Equity	1.20	0.15
Mean		**0.89**	**0.17**
Median		**0.87**	**0.17**

Source: Bloomberg and Frontier Economics analysis.

Table 7: Re-levered equity betas using weekly data

Name	Ticker	Monday	Tuesday	Wednesday	Thursday	Friday	Mean re-levered equity beta	Standard deviation over reference days
AMERICAN STS.WATER	AWR US Equity	1.12	1.05	0.86	1.02	0.89	0.99	0.11
AMERICAN WATER WORKS	AWK US Equity	0.94	0.79	0.64	0.76	1.01	0.83	0.15
ARTESIAN RES.'A'	ARTNA US Equity	0.84	0.93	0.89	0.80	1.01	0.89	0.08
CAL.WATER SER.	CWT US Equity	0.91	0.93	0.96	0.99	0.87	0.93	0.05
CONNECTICUT WATER	CTWS US Equity	0.55	0.64	0.74	0.64	0.55	0.62	0.08
ESSENTIAL UTILITIES	WTRG US Equity	1.33	1.12	1.05	1.19	1.26	1.19	0.11
MIDDLESEX WATER	MSEX US Equity	1.07	1.16	1.17	1.11	1.20	1.14	0.05
PENNON GROUP	PNN LN Equity	0.78	0.67	0.57	0.77	0.72	0.70	0.08
SEVERN TRENT	SVT LN Equity	0.73	0.66	0.52	0.68	0.71	0.66	0.08
SJW GROUP	SJW US Equity	0.98	0.93	1.00	0.98	1.19	1.02	0.10
UNITED UTILITIES GROUP	UU/ LN Equity	0.76	0.67	0.53	0.68	0.68	0.67	0.08
YORK WATER	YORW US Equity	1.35	1.29	1.18	1.42	1.54	1.36	0.14
Mean		**0.95**	**0.90**	**0.84**	**0.92**	**0.97**	**0.92**	**0.09**
Median		**0.93**	**0.93**	**0.88**	**0.89**	**0.95**	**0.91**	**0.08**

Source: Bloomberg and Frontier Economics analysis.

79. Table 7 above provides similar estimates at the weekly frequency, with individual reference day results included.

80. Applying an equal weighting to the weekly and monthly average estimates, yields an estimate of 0.92 for the re-levered equity beta.[39] This is materially higher than the current Incenta estimate of 0.77. Even the lowest estimate, 0.89 (the mean of monthly beta estimates), is materially above 0.77.

81. The analysis further highlights the imprecision caused by using a single reference day. If weekly estimates are obtained using Friday as the reference day, an average of 0.97 would be obtained. If instead Wednesday was arbitrarily chosen as the reference day the beta estimate would be 0.84. This highlights the fact that a variation in beta estimates of 0.09 (resulting from my update of the Incenta approach) is immaterial relative to the magnitude of variation that can result simply from defining the sampling period to start on a different day of the week.

4.2 Expanded comparator set

82. Taking the expanded comparator set obtained by examining the activities of comparators identified by a number of classifications schemes, I obtain beta estimates for 21 comparators that pass the liquidity filters. As above, I obtain the means over reference days for each comparator and returns frequency. The results are presented in Table 8 below.[40]

83. The resulting beta estimates are even higher when this expanded sample is used. Weighting weekly and monthly average estimates yields an estimate of 0.95; using median estimates yields 0.87. Both are materially above 0.77.

[39] A figure of 0.90 is obtained if the medians are used.

[40] Note that the results for Incenta comparators may differ from **Table 6** and **Table 7** as the Vasicek adjustment depends on the raw estimates of other comparators.

Table 8: Re-levered equity beta estimates – expanded comparator set

Name	Country	Ticker	Mean weekly	Mean monthly
AGUAS ANDINAS	Chile	AGUAS/A CI Equity	1.04	1.12
AMERICAN STS.WATER	United States	AWR US Equity	0.92	0.78
AMERICAN WATER WORKS	United States	AWK US Equity	0.79	0.63
ARTESIAN RES.'A'	United States	ARTNA US Equity	0.78	0.83
CAL.WATER SER.	United States	CWT US Equity	0.88	0.79
CHINA WATER AFFAIRS GP.	Hong Kong	855 HK Equity	0.80	0.84
CONNECTICUT WATER	United States	CTWS US Equity	0.52	0.48
CONSOLIDATED WT.	United States	CWCO US Equity	2.11	1.89
CPAD.SANMT.BASICO DE SAOP.ON	Brazil	SBSP3 BZ Equity	1.32	1.35
CPAD.SANMT.DE MINASGR.ON	Brazil	CSMG3 BZ Equity	1.13	1.23
ESSENTIAL UTILITIES	United States	WTRG US Equity	1.16	1.17
INVERSIONES AGUAS METROPOLITANAS	Chile	IAM CI Equity	0.77	0.85
MANILA WATER	Philippines	MWC PM Equity	0.73	0.88
MIDDLESEX WATER	United States	MSEX US Equity	1.10	0.97
PENNON GROUP	United Kingdom	PNN LN Equity	0.67	0.67
SEVERN TRENT	United Kingdom	SVT LN Equity	0.63	0.68
SJW GROUP	United States	SJW US Equity	0.97	1.04
SUEZ	France	SEV FP Equity	0.82	0.88
THAI TAP WATER SUPPLY	Thailand	TTW TB Equity	0.85	0.91
UNITED UTILITIES GROUP	United Kingdom	UU/ LN Equity	0.64	0.68
YORK WATER	United States	YORW US Equity	1.29	1.09
Mean			**0.95**	**0.94**
Median			**0.85**	**0.88**

Source: Bloomberg and Frontier Economics analysis.

4.3 Comparator set drawn from developed economies only

84. In order to check whether my results are driven by the inclusion of comparators from emerging (and therefore potentially risky) markets, I investigate the impact of considering only those comparators drawn from developed economies (as defined by MSCI).

85. Taking the expanded comparator set obtained above, and restricting to developed markets, I obtain beta estimates for 15 comparators that pass the liquidity filters.[41] As above, I obtain the means over reference days for each comparator and returns frequency. The results are presented in Table 9 below.[42]

[41] These comparators are in the Hong Kong, France, United Kingdom and United States markets.

[42] Note that the results for comparators may differ from **Table 6**, **Table 7** and **Table 8** as the Vasicek adjustment depends on the raw estimates of other comparators.

Table 9: Re-levered equity beta estimates – developed economy comparator set

Name	Country	Ticker	Mean weekly	Mean monthly
AMERICAN STS.WATER	United States	AWR US Equity	0.92	0.79
AMERICAN WATER WORKS	United States	AWK US Equity	0.79	0.64
ARTESIAN RES.'A'	United States	ARTNA US Equity	0.78	0.84
CAL.WATER SER.	United States	CWT US Equity	0.88	0.80
CHINA WATER AFFAIRS GP.	Hong Kong	855 HK Equity	0.80	0.84
CONNECTICUT WATER	United States	CTWS US Equity	0.52	0.49
CONSOLIDATED WT.	United States	CWCO US Equity	2.12	1.89
ESSENTIAL UTILITIES	United States	WTRG US Equity	1.16	1.17
MIDDLESEX WATER	United States	MSEX US Equity	1.10	0.98
PENNON GROUP	United Kingdom	PNN LN Equity	0.67	0.67
SEVERN TRENT	United Kingdom	SVT LN Equity	0.63	0.68
SJW GROUP	United States	SJW US Equity	0.97	1.05
SUEZ	France	SEV FP Equity	0.82	0.88
UNITED UTILITIES GROUP	United Kingdom	UU/ LN Equity	0.64	0.68
YORK WATER	United States	YORW US Equity	1.28	1.10
Mean			**0.94**	**0.90**
Median			**0.82**	**0.84**

Source: Bloomberg and Frontier Economics analysis.

86. The resulting beta estimates are similar to those of the Incenta comparators, and remarkably consistent with the estimates derived using the full, expanded sample of 21 comparators drawn from developed and emerging economies. Weighting weekly and monthly average estimates yields an estimate of 0.92; using median estimates yields 0.83. Both are materially above 0.77.

5 Conclusions on equity beta

87. I have updated the point and upper bound estimates, following the Incenta approach, to reflect the most recently available data. I find that neither the point estimate nor the upper bound have changed materially since the QCA's last review. The point estimate is 9 basis point lower and the upper bound is 24 basis points higher. Both changes are statistically insignificant – being tiny relative to the degree of statistical estimation error involved in this process. That is, these changes in estimates are highly likely due to random statistical estimation error, rather than a real shift in the true systematic risk of the comparator firms.

88. I also identify a number of potential improvements to the approach currently adopted by Incenta – designed to improve the robustness of equity beta estimates and to reduce the impact of random statistical estimation error. I show that the adoption of these improvements to the beta estimation process currently have the effect of increasing the equity beta estimates for water businesses above 0.77.

89. In summary, I note that the evidence does not support a reduction in the equity beta, but rather the most robust evidence suggests that, if any change is to be made, the allowed equity beta should be increased. In my view, the evidence currently supports a re-levered equity beta of at least 0.9.

A Derivation of de-levering and re-levering formulas with constant gearing

Setting

This Appendix contains a full derivation of formulas for the relationship between asset and equity betas for a firm with a constant leverage ratio. I show that under the assumption of constant gearing, the de-levering and re-levering formulas do not contain the tax rate term.

Consider a firm with a target leverage ratio of L and assume that the firm continuously adjusts its capital structure to maintain this leverage ratio at all times. That is $\frac{D_i}{V_i} = L$ for all points in time, *i*. No assumptions are made about the pattern of the firm's operating cash flows.

The effect of tax benefits on the WACC

First note that if the firm were unlevered, its value would be given by:

$$V^U = \sum_{i=1}^{N} \frac{C_i(1-\tau)}{(1+r_a)^i}, \qquad (1)$$

where:

- V^U is the value of an otherwise identical unlevered firm;
- τ is the effective corporate tax rate; and
- r_a is the required return on the assets of the firm, which is the same as the return required by equity holders in an otherwise identical unlevered firm.

Under the CAPM, the required return on the assets of the firm is:

$$r_a = r_f + \beta_a \ MRP, \qquad (2)$$

where:

- r_f is the risk-free rate of interest;
- β_a is the asset beta or systematic risk of the firm's assets. This is the same as the equity beta for an otherwise identical unlevered firm; and
- *MRP* is the market risk premium--the expected return on the market portfolio in excess of the risk-free rate.

The after-tax operating cash flows for an otherwise identical levered firm exceed those of the unlevered firm by the tax savings due to interest payments. Thus, the cash flows in period *i* are:

$$C_i(1-\tau) + r_d D_{i-1}\tau, \qquad (3)$$

where:

- r_d is the required return on debt; and
- D_{i-1} is the value of debt at time *i - 1*.

The first component of this cash flow is identical to that of the unlevered firm, and therefore should be discounted at r_a. The second component must be discounted at a rate that reflects the risk of tax benefits. Note that at Time *i - 1* the value of debt, D_{i-1}, is known and the tax benefit at time *i*, $r_d D_{i-1}\tau$, is a fixed multiple of the required return on debt, r_d. Thus, it is appropriate to discount the tax benefit from time *i* to time *i - 1* using r_d.

Note that we can't use r_d to discount these tax benefits right back to time 0 because the amount of debt varies over time with the cash flows of the firm. We can only use r_d to discount from time *i* to *i-1* because the tax saving at time *i* depends on the amount of debt at time *i-1*, D_{i-1}, which is only known at time *i-1*.

Now, we use induction (recursively) to determine the relationship between r_*^L and r_a. First note that at time *N*, the firm generates its last cash flow of $C_N(1-\tau)$. One period before this (at time *N - 1*, immediately *after* the cash flow at *N - 1* has been paid) the value of the unlevered firm is:

$$V_{N-1}^U = \frac{C_N(1-\tau)}{1+r_a}.$$

The value of the otherwise identical levered firm at the same time is:

$$V_{N-1}^L = \frac{C_N(1-\tau)}{1+r_a} + \frac{r_d D_{N-1}\tau}{1+r_d}.$$

Note that $D_{N-1} = L\, V_{N-1}^L$ since the firm has a constant leverage ratio of *L*. Therefore:

$$V_{N-1}^L = \frac{C_N(1-\tau)}{1+r_a} + \frac{r_d L V_{N-1}^L \tau}{1+r_d}.$$

So,

$$V_{N-1}^L \left[1 - \frac{r_d L\tau}{1+r_d}\right] = \frac{C_N(1-\tau)}{1+r_a},$$

and,

$$V_{N-1}^L = \frac{C_N(1-\tau)}{(1+r_a)\left(1-\frac{r_d L\tau}{1+r_d}\right)}. \quad (4)$$

Thus, when discounting from time *N* to time *N - 1*, we must use:

$$(1+r_*^L) = (1+r_a)\left(1-\frac{r_d L\tau}{1+r_d}\right)$$
$$= 1 + r_a - \frac{r_d L\tau(1+r_a)}{1+r_d}.$$

Consequently,

$$r_*^L = r_a - r_d L\tau\frac{1+r_a}{1+r_d}.$$

Note that at time *N - 2*, the value of the levered firm is:

$$V_{N-2}^L = \frac{C_{N-1}(1-\tau)}{1+r_a} + \frac{r_d L\, V_{N-2}^L\tau}{1+r_d} + \frac{V_{N-1}^L}{1+r_a}.$$

Now we substitute for V_{N-1}^L in (4):

$$V_{N-2}^{L} = \frac{C_{N-1}(1-\tau)}{1+r_a} + \frac{r_d L \, V_{N-2}^{L}\tau}{1+r_d} + \frac{C_N(1-\tau)}{(1+r_a)^2\left(1-\frac{r_d L\tau}{1+r_d}\right)}.$$

So,

$$V_{N-2}^{L}\left(1-\frac{r_d L\tau}{1+r_d}\right) = \frac{C_{N-1}(1-\tau)}{1+r_a} + \frac{C_N(1-\tau)}{(1+r_a)^2\left(1-\frac{r_d L\tau}{1+r_d}\right)}.$$

Also,

$$\begin{aligned} V_{N-2}^{L} &= \frac{C_{N-1}(1-\tau)}{(1+r_a)\left(1-\frac{r_d L\tau}{1+r_d}\right)} + \frac{C_N(1-\tau)}{(1+r_a)^2\left(1-\frac{r_d L\tau}{1+r_d}\right)^2} \\ &= \frac{C_{N-1}(1-\tau)}{1+r_*^L} + \frac{C_N(1-\tau)}{(1+r_*^L)^2}, \end{aligned}$$

where,

$$1+r_*^L = (1+r_a)\left(1-\frac{r_d L\tau}{1+r_d}\right).$$

We can do the same thing recursively from *N - 2* to *N - 3* and so on. In all cases, we have:

$$r_*^L = r_a - r_d L\tau\frac{1+r_a}{1+r_d}. \tag{5}$$

Note that this implies:

$$\begin{aligned} 1+r_*^L &= 1+r_a - \frac{r_d L\tau}{1+r_d}(1+r_a) \\ &= (1+r_a)\left(1-\frac{r_d L\tau}{1+r_d}\right) \\ &= \frac{(1+r_a)(1+r_d-r_d L\tau)}{1+r_d} \\ &= \frac{(1+r_a)(1+r_d(1-L\tau))}{1+r_d}. \end{aligned}$$

Constant re-balancing to the Target Leverage Ratio

Next, note that this expression simplifies considerably if the time between each period becomes arbitrarily small. If, for example, interest compounds *m* times per year, we have the following relation:

$$\left(1+\frac{r_*^L}{m}\right)^m = \left(\frac{\left(1+\frac{r_a}{m}\right)\left(1+\frac{r_d(1-L\tau)}{m}\right)}{\left(1+\frac{r_d}{m}\right)}\right)^m.$$

Taking the limit as *m* becomes large gives:

$$\lim_{m\to\infty}\left(1+\frac{r_*^L}{m}\right)^m = \lim_{m\to\infty}\left(\frac{\left(1+\frac{r_a}{m}\right)\left(1+\frac{r_d(1-L\tau)}{m}\right)}{\left(1+\frac{r_d}{m}\right)}\right)^m,$$

which implies that:

$$e^{r_*^L} = e^{r_a+r_d(1-L\tau)-r_d} = e^{r_a-r_d L\tau}.$$

Consequently, if we assume that the firm *continuously* rebalances its capital structure to the target leverage ratio, *L*, the relationship between the cost of capital of a levered firm and an otherwise identical unlevered firm is:

$$r_*^L = r_a - r_d L\tau,$$

or,

$$r_*^L = r_a - r_d \tau \frac{D}{V} \tag{6}$$

Classic Weighted-average Cost of Capital

The cash flow to equity at any time *N* can be written as:

$$E_N^L = C_N(1-\tau) - r_d D_{N-1} + \tau r_d D_{N-1} + (D_N - D_{N-1}) + (1-L)V_N^L.$$

That is, the equity holders receive the after-tax operating cash flow, $C_N(1-\tau)$, less the interest paid to debtholders, $r_d D_{N-1}$. They also receive the tax benefit on interest $\tau r_d D_{N-1}$ plus any net change in the amount of debt, $D_N - D_{N-1}$. If the amount of debt financing increases $(D_N > D_{N-1})$, this additional cash is available to the equity holders and vice versa. They also receive a proportion $(1-L)$ of the present value of all future cash flows which is V_N^L at time *N*.

Substituting $D_N = LV_N^L$ yields:

$$\begin{aligned} E_N^L &= C_N(1-\tau) - r_d LV_{N-1}^L + \tau r_d LV_{N-1}^L + LV_N^L - LV_{N-1}^L + (1-L)V_N^L \\ &= C_N(1-\tau) + V_N^L - [1 + r_d(1-\tau)]LV_{N-1}^L. \end{aligned}$$

Dividing all terms by the value of equity at *N - 1* gives:

$$\frac{E_N^L}{E_{N-1}^L} = \frac{C_N(1-\tau) + V_N^L}{(1-L)V_{N-1}^L} - \frac{[1 + r_d(1-\tau)]LV_{N-1}^L}{(1-L)V_{N-1}^L}$$

which implies that:

$$1 + r_e = \frac{1 + r_*^L}{(1-L)} - [1 + r_d(1-\tau)]\frac{L}{1-L},$$

because $V_{N-1}^L(1 + r_*^L) = C_N(1-\tau) + V_N^L$. That is, the value of the whole firm must increase to provide a return of r_*^L over the period.

This implies that:

$$(1 + r_e)(1-L) = 1 + r_*^L - [1 + r_d(1-\tau)]L.$$

So,

$$1 + r_*^L = 1 + r_e(1-L) + [1 + r_d(1-\tau)]L,$$

and,

$$r_*^L = r_e(1-L) + r_d(1-\tau)L,$$

or,

$$r_*^L = r_e \frac{E}{V} + r_d(1-\tau)\frac{D}{V}, \tag{7}$$

which is the standard expression for the 'classic' definition of WACC.

Cost of Equity

Rearranging the WACC equation yields:

$$r_e = \frac{V}{E} r_*^L - r_d(1-\tau)\frac{D}{E}.$$

Recall that if the capital structure is continuously rebalanced to the target leverage ratio:

$$r_*^L = r_a - r_d \tau \frac{D}{V}.$$

So,

$$\begin{aligned} r_e &= \frac{V}{E}\left(r_a - r_d\tau\frac{D}{V}\right) - r_d(1-\tau)\frac{D}{E} \\ &= \frac{E}{E} r_a + \frac{D}{E} r_a - \frac{D}{E} r_d \tau - r_d(1-\tau)\frac{D}{E} \\ &= r_a + \frac{D}{E} r_a - \frac{D}{E} r_d \tau - \frac{D}{E} r_d + \frac{D}{E} r_d \tau. \end{aligned} \qquad (8)$$

So,

$$r_e = r_a + (r_a - r_d)\frac{D}{E}.$$

Levering and Un-levering Betas

Finally, we need an expression to relate the equity beta of a levered firm to the asset beta (or equity beta of an otherwise identical unlevered firm). Combining Equations (5) and (8) yields:

$$r_*^L = r_a - r_d\tau\frac{D}{V} = r_e\frac{E}{V} + r_d(1-\tau)\frac{D}{V}.$$

Now, substituting expressions for r_a, r_e, and r_d from the CAPM yields:

$$(r_f + \beta_a MRP) - (r_f + \beta_d MRP)\tau\frac{D}{V} = (r_f + \beta_e MRP)\frac{E}{V} + (r_f + \beta_d MRP)(1-\tau)\frac{D}{V}.$$

This implies that:

$$r_f\left(1 - \tau\frac{D}{V}\right) + \left(\beta_a - \beta_d\tau\frac{D}{V}\right)MRP = r_f\left(\frac{E}{V} + \frac{D}{V} - \tau\frac{D}{V}\right) + \left(\beta_e\frac{E}{V} + \beta_d(1-\tau)\frac{D}{V}\right)MRP.$$

So,

$$\begin{aligned} \beta_a - \beta_d\tau\frac{D}{V} &= \beta_e\frac{E}{V} + \beta_d(1-\tau)\frac{D}{V} \\ \beta_a &= \beta_e\frac{E}{V} + \beta_d\left[\frac{D}{V} - \tau\frac{D}{V} + \tau\frac{D}{V}\right]. \end{aligned}$$

So,

$$\beta_a = \beta_e\frac{E}{V} + \beta_d\frac{D}{V}. \qquad (9)$$

This implies that:

$$\beta_e = \beta_a \frac{V}{E} - \beta_d \frac{D}{V}\frac{V}{E}$$
$$\beta_e = \beta_a \left(1 + \frac{D}{E}\right) - \beta_d \frac{D}{E}.$$

So,

$$\beta_e = \beta_a + (\beta_a - \beta_d)\frac{D}{E}. \quad (10)$$

That is, when a constant leverage ratio is assumed, then the relevant de-levering and re-levering formulas contain no tax rate term.

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