

**ESTIMATION OF THE UTILISATION RATE ON IMPUTATION CREDITS
BY THE DIVIDEND DROP-OFF METHODOLOGY**

Report prepared for the Queensland Competition Authority

Martin Lally

Associate Professor

School of Economics and Finance

Victoria University of Wellington

April 18 2005

1. Introduction

Implementation of the Officer (1995) version of the Capital Asset Pricing Model requires, inter alia, an estimate of the utilisation rate on imputation credits (U). One of the many methods for estimating this parameter is through an examination of dividend drop-off ratios on dividend ex-days. Christensen (2004) adopts this approach, and in particular compares the results from 1/1/1990 - 30/6/2000 (period 1) with those from 30/6/2000 – 31/3/2004 (period 2), with the split corresponding to the introduction of regulations restricting the trading of imputation credits. Christensen finds that the estimate for U is lower in period 2, and close to zero in this period (.04), and he advances this latter estimate¹. He does not specify the standard error on the estimate or the difference in estimates between the two periods, and it therefore not apparent whether the reduction is statistically significant or how much reliance can be placed upon his estimate for period 2.

Lally (2004) notes a number of general concerns with estimates of U derived from dividend ex-day studies. Firstly, the estimate will presumably reflect the presence of foreign investors in the Australian market, and this is inconsistent with the use of a domestic version of the CAPM such as the Officer model. Secondly, ex-dividend day returns are known to exhibit perverse behaviour, which contaminates the estimate (see, for example, Frank and Jagannathan, 1998, in respect of Hong Kong and Brown and Walter, 1986, in respect of Australia). Thirdly, ex-dividend day returns may reflect the actions of arbitrageurs in a particular tax bracket buying just before and selling just after the ex-day, and this also contaminates the estimate of U . Finally, the statistical reliability of the estimates from dividend ex-day studies tends to be poor. Of these points, only the last requires elaboration in respect of Christensen's study, and this is the objective of the present paper.

Section 2 examines the regression model invoked by Christensen to estimate U , and shows that Christensen has incorrectly estimated U from the regression parameters.

¹ Christensen refers throughout his study to “gamma” rather than the utilisation rate (U), with “gamma” being defined as the proportion of company taxes that are claimed as a credit against investors' personal tax obligations. However, as defined in this way, “gamma” is the product of U and the proportion of company taxes that are assigned by the company to its dividends. The drop-off methodology employed by him can only estimate U , not gamma.

The correct estimates of U are then derived for periods 1 and 2. Section 3 then considers the question of deleting outliers in the regression, and shows that the estimate for period 2 is very sensitive to such deletions. Section 4 then determines the 95% confidence intervals on the estimates for the two periods and on the difference in the estimates for the two periods. It is shown that the confidence intervals on the individual estimates are very large and the difference in the estimates for periods 1 and 2 is not statistically significant.

2. The Regression model

Christensen conducts the following regression (ibid, p 46)

$$\frac{P_1 - P_2}{DIV} = \alpha + \beta \left[\frac{ft}{1-t} \right] + e \quad (1)$$

where P_1 is the share price before the ex-time, P_2 is the share price after it, DIV is the cash dividend, t is the corporate tax rate, f is the franking proportion (1 for fully franked dividends and 0 for unfranked dividends) and e is the regression residual. The regression process generates estimates for α and β , denoted $\hat{\alpha}$ and $\hat{\beta}$. Christensen's estimate of the estimated utilisation rate is then as follows.

$$\hat{U} = \frac{\hat{\beta}}{1-t} \quad (2)$$

Christensen's estimate for β for period 2 is .025, after deletion of some outlier observations. Substitution of this into equation (2), along with the relevant company tax rate of $t = .30$, yields his estimate for U of .04.

Although Christensen does not derive this transformation formula (2), he nonetheless uses it. However it is incorrect, and the correct formula is as follows. In equilibrium, the expected change in share price net of the capital gains tax should be equal to the dividend net of the tax effects, i.e.,

$$E(P_1 - P_2)(1 - t_g) = DIV \left[1 + U \frac{IC}{DIV} \right] (1 - t_p) \quad (3)$$

where t_g is the tax rate on capital gains, t_p is the tax rate on ordinary income and IC are the imputation credits attached to the dividend. The imputation credits cannot exceed proportion $t/(1 - t)$ of the cash dividend. So, the franking proportion is then as follows.

$$f = \frac{IC}{DIV \left[\frac{t}{1-t} \right]}$$

Thus

$$IC = DIV f \left[\frac{t}{1-t} \right]$$

Substitution of this into equation (3) yields

$$E(P_1 - P_2)(1 - t_g) = DIV \left[1 + \frac{Uft}{1-t} \right] (1 - t_p)$$

Rearrangement yields

$$\frac{E(P_1 - P_2)}{DIV} = \frac{1 - t_p}{1 - t_g} + U \left[\frac{1 - t_p}{1 - t_g} \right] f \left[\frac{t}{1-t} \right] \quad (4)$$

Assuming (reasonably) that the expected decline in share price is unbiased, it follows that

$$P_1 - P_2 = E(P_1 - P_2) + u$$

where u is a mean zero residual. Substitution into equation (4) yields

$$\frac{P_1 - P_2}{DIV} = \frac{1 - t_p}{1 - t_g} + U \left[\frac{1 - t_p}{1 - t_g} \right] f \left[\frac{t}{1-t} \right] + \frac{u}{DIV}$$

Comparison with Christensen's regression model in equation (1) reveals that

$$\alpha = \frac{1-t_p}{1-t_g}, \quad \beta = U \left[\frac{1-t_p}{1-t_g} \right] = U\alpha$$

So the estimated utilisation rate is then as follows.

$$\hat{U} = \frac{\hat{\beta}}{\hat{\alpha}} \tag{5}$$

Equation (5) differs from Christensen's transformation equation (2). In particular, Christensen fails to divide through by $\hat{\alpha}$ and instead divides through by the tax term $(1-t)$.

Invoking the transformation equation (5), and Christensen's estimates for α and β in period 2 of .908 and .025 respectively, the correct estimate for U for period 2 is then

$$\hat{U} = \frac{.025}{.908} = .028$$

In respect of period 1, Christensen provides regression results for three subperiods rather than for this period in aggregate². So, the regression is conducted for period 1 in aggregate, using Christensen's data set³. This generates estimates for α and β of .720 and .114 respectively. Substitution of these into the transformation equation (5) yields an estimate for U in period 1 of

$$\hat{U} = \frac{.114}{.720} = .158$$

² This presumably occurs because Christensen's equation (2) involves the company tax rate, and this has changed over time.

³ Consistent with Christensen's approach, observations with zero values for the drop-off ratio are deleted (on the grounds that they are likely to be measurement errors). In addition, all observations whose drop-off ratios diverge from the mean drop-off ratio by more than 10 are deleted. If these latter deletions are not made then the resulting estimates of α and β are .720 and .077, yielding an estimate for U of .107 rather than the estimate of .158 noted above.

3. The Effect of Deleting Outliers

Christensen deletes a number of outliers from his regressions, comprising zero values and extreme values for the dependent variable (the drop-off ratio). The deletion of the extreme values has a very significant effect upon the estimate of β , and hence U , for period 2⁴. Had Christensen not deleted these outliers, the estimates of α and β would have been .542 and .782 rather than .908 and .025 respectively. Following equation (5), the resulting estimate of U would then have been 1.443 rather than .028 above⁵. Christensen's claim that U is close to zero in period 2 would then be invalid, and the point estimate would also be implausible (U must lie between 0 and 1). Deleting outliers is a perfectly common and respectable practice, but there are no objective rules for it. As a consequence, the results must be treated with considerable caution, especially given their sensitivity in this case to the rule applied for deleting outliers.

Christensen deletes values for the dependent variable Y that exceed 8 or are less than -8 . Given that the average value for the variable is not zero (it is .78), this is a curious choice. A more natural basis for exclusion would be values that were more than some quantity away from .78, i.e., $.78 \pm x$. Table 1 reports the results of doing so, with the second column showing the proportion of observations that are excluded.

Table 1: The Effect of Excluding Outliers

Exclusion Rule	Prop	$\hat{\alpha}$	$\hat{\beta}$	\hat{U}
No exclusions	0	.542	.782	1.443
$Y \leq -19.22, Y \geq 20.78$	0.5%	.856	.130	.152
$Y \leq -9.22, Y \geq 10.78$	1%	.966	-.108	-.112
$Y \leq -6.22, Y \geq 7.78$	1.7%	.962	-.067	-.070
$Y \leq -3.22, Y \geq 4.78$	4%	.899	.080	.089
$Y \leq -1.22, Y \geq 2.78$	10%	.871	.198	.227

⁴ The effect of deleting outliers in the first period is much less pronounced, as noted in footnote 3.

⁵ Using Christensen's equation (2), the result would have been 1.12 rather than .025.

Whilst one cannot say which of these exclusion rules should apply, the rules in the second and third rows are at least plausible because they involve a small proportion of deletions. However, the resulting estimates of U range from .152 to -.112 (or zero if negative values for U are rejected as not plausible). All of this suggests that the dividend drop-off methodology for estimating U is unsatisfactory.

4. Confidence Intervals

The first step in determining a confidence interval is in determining the variance of the estimate. Equation (5) implies that

$$\text{Var}(\hat{U}) = \text{Var}\left(\frac{\hat{\beta}}{\hat{\alpha}}\right)$$

Rice (1995, p 153) gives the following approximation for the variance of the ratio of two random variables.

$$\text{Var}\left(\frac{\hat{\beta}}{\hat{\alpha}}\right) = \frac{1}{E^2(\hat{\alpha})} \left[\text{Var}(\hat{\alpha}) \frac{E^2(\hat{\beta})}{E^2(\hat{\alpha})} + \text{Var}(\hat{\beta}) - 2\text{Cov}(\hat{\alpha}, \hat{\beta}) \frac{E(\hat{\beta})}{E(\hat{\alpha})} \right]$$

It follows from the last two equations that

$$\text{Var}(\hat{U}) = \frac{1}{E^2(\hat{\alpha})} \left[\text{Var}(\hat{\alpha}) \frac{E^2(\hat{\beta})}{E^2(\hat{\alpha})} + \text{Var}(\hat{\beta}) - 2\text{Cov}(\hat{\alpha}, \hat{\beta}) \frac{E(\hat{\beta})}{E(\hat{\alpha})} \right] \quad (6)$$

In respect of Christensen's regression for period 2, the sample estimates of these variances and means are as follows.

$$\hat{\alpha} = .908$$

$$\hat{\beta} = .025$$

$$\hat{\text{Var}}(\hat{\alpha}) = .080^2$$

$$\hat{V}ar(\hat{\beta}) = .220^2$$

In respect of the estimated covariance, this is as follows

$$Cov(\hat{\alpha}, \hat{\beta}) = \frac{-\bar{X}}{\sum (X_i - \bar{X})^2} \left[\frac{\sum e_i^2}{n-2} \right] = \frac{-\bar{X}\hat{\beta}^2}{\sum (Y_i - \bar{Y})^2 - \sum e_i^2} \left[\frac{\sum e_i^2}{n-2} \right] \quad (7)$$

where X_i are the observed values of the independent variable in the regression, \bar{X} is their sample mean, Y_i are the observed values of the dependent variable in the regression, \bar{Y} is their sample mean, e_i are the residuals around the estimated regression line and n is the sample size (Johnston, 1972, pp. 21-36). Substituting from Christensen's regression results into equation (7) yields the following.

$$C\hat{o}v(\hat{\alpha}, \hat{\beta}) = \frac{-.30(.025)^2}{.027} [2.06] = -.014$$

Substitution of these results into equation (6) yields the following result for period 2.

$$Var(\hat{U}_2) = .243^2$$

In respect of the data for period 1, the sample estimates of the variances and means are as follows.

$$\hat{\alpha} = .720$$

$$\hat{\beta} = .114$$

$$\hat{V}ar(\hat{\alpha}) = .067^2$$

$$\hat{V}ar(\hat{\beta}) = .139^2$$

Substituting from the regression results into equation (7) yields the following.

$$C\hat{o}v(\hat{\alpha}, \hat{\beta}) = \frac{-.394(.114)^2}{1.697} [2.512] = -.008$$

Substitution of these results into equation (6) yields the following result for period 1.

$$\text{Var}(\hat{U}_1) = .206^2$$

The 95% confidence interval for the point estimate for period 2 is then approximately

$$.028 \pm (.243)2 = .028 \pm .486$$

For period 1 the result is

$$.158 \pm (.206)2 = .158 \pm .412$$

These confidence intervals are so wide that little reliance can be placed upon the individual point estimates of .028 and .158 for the two periods.

In respect of the difference between the point estimates of U for the two periods, the independence of the estimates implies that the variance of the difference is as follows.

$$\text{Var}(\hat{U}_1 - \hat{U}_2) = \text{Var}(\hat{U}_1) + \text{Var}(\hat{U}_2)$$

Substituting the results above yields the following

$$\text{Var}(\hat{U}_1 - \hat{U}_2) = .206^2 + .243^2 = .319^2$$

So, the 95% confidence interval on the *difference* in the estimates of U is then

$$.158 - .028 \pm (.319)2 = .130 \pm .638$$

This confidence interval easily embraces zero. So, the difference between the estimates of U is not statistically significant. In fact, given that the standard deviation on the difference in point estimates is .319, the point estimates would have to differ by at least .638 before the difference was statistically significant.

5. Conclusion

Estimation of U by examination of ex-dividend day returns suffers from a number of difficulties. Firstly, the estimate will presumably reflect the presence of foreign investors in the Australian market, and this is inconsistent with the use of a domestic version of the CAPM such as the Officer model. Secondly, ex-dividend day returns are known to exhibit perverse behaviour, which contaminates the estimate. Thirdly, ex-dividend day returns may reflect the actions of arbitrageurs in a particular tax bracket buying just before and selling just after the ex-day, and this also contaminates the estimate. Finally, the statistical reliability of the estimates tends to be low and therefore caution is warranted in placing any reliance upon them.

This paper investigates the last issue in respect of Christensen's (2004) paper. His estimates of U based upon the results of a regression model are incorrect, although the effect is only moderate. In addition, the estimate for the period since mid 2000 is very sensitive to the rule applied for deleting outliers. Furthermore the confidence intervals for the estimates before and after legislative changes in mid 2000 that may have lowered the value of U are very wide. Finally, the difference in the estimates of U before and after this legislative change is not statistically significant.

In view of all this, the point estimate for U of .028 for the period since mid 2000, arising from the regression methodology and Christensen's selection of outliers for deletion, should be treated with considerable scepticism. Alternative bases for selection of the outliers still yields estimates that should be viewed in the same way.

REFERENCES

Brown, P. and Walter, T. 1986, 'Ex-Dividend Day Behaviour of Australian Share Prices', *Australian Journal of Management*, vol. 11, pp. 139-52.

Christensen, M. 2004, *Review of Cost of Capital Inputs for the 2005 Review of QR's Access Undertaking*, paper prepared for Queensland Rail.

Frank, M. and Jagannathan, R. 1998, 'Why do Stocks Prices Drop by less than the Value of the Dividend? Evidence from a Country without Taxes', *Journal of Financial Economics*, vol. 47, pp. 161-88.

Johnston, J. 1972, *Econometric Methods*, McGraw-Hill (New York).

Lally, M. 2004, *The Cost of Capital for Regulated Entities*, paper prepared for the Queensland Competition Authority.

Officer, R. 1994, 'The Cost of Capital of a Company under an Imputation Tax System', *Accounting and Finance*, vol. 34, pp. 1-17.

Rice, J. 1995, *Mathematical Statistics and Data Analysis*, 2nd edition, Duxbury Press (Belmont, California).