The Relationship Between Franking Credits and the Market Risk Premium: A Review

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1. Introduction

SFG (2004) have recently argued that the QCA's adoption of a 6% estimate for the market risk premium in the Officer version of the CAPM is inconsistent with their adoption of a "gamma" value of .50. In particular, at this gamma value, and with a risk free rate of 5.84%, SFG argues that the appropriate estimate for the market risk premium is 7.1 – 10%. This paper seeks to review these arguments. Section 2 presents the arguments. Section 3 evaluates the arguments and identifies two significant errors in the analysis. Finally, section 4 assesses the implications of the "consistency" principle for the QCA's conclusions about the market risk premium.

2. The Argument

Under the Officer (1994) model, the cost of equity is defined to comprise dividends inclusive of imputation credits (to the extent of being usable) along with capital gains and is as follows

$$\hat{k}_e = R_f + [\hat{k}_m - R_f]\beta_e$$

where R_f is the risk free rate, $\hat{k}_m$ is the expected return on the market portfolio (inclusive of dividend imputation credits to the extent of being usable), and β_e the equity beta. The market risk premium is the term [ ]. SFG assert that the proportion of $\hat{k}_m$ that is attributable to imputation credits is

$$\frac{\gamma\tau}{1-\tau(1-\gamma)}$$

Accordingly they decompose $\hat{k}_m$ as follows

$$\hat{k}_m = \hat{k}_m\left[\frac{1-\tau}{1-\tau(1-\gamma)}\right] + \hat{k}_m\left[\frac{\gamma\tau}{1-\tau(1-\gamma)}\right]$$

Subtracting R_f from both sides then yields

$$\hat{k}_m - R_f = \left\{ \hat{k}_m \left[\frac{1-\tau}{1-\tau(1-\gamma)} \right] - R_f \right\} + \hat{k}_m \left[\frac{\gamma\tau}{1-\tau(1-\gamma)} \right]$$

or

$$M\hat{R}P = \left\{ [M\hat{R}P + R_f] \left[\frac{1-\tau}{1-\tau(1-\gamma)} \right] - R_f \right\} + [M\hat{R}P + R_f] \left[\frac{\gamma\tau}{1-\tau(1-\gamma)} \right] \qquad (1)$$

The term $M\hat{R}P$ on the left hand side of equation (1) is the market risk premium in the Officer model, the term { } on the right hand side represents the part of the market risk premium comprising cash dividends and capital gains, and the remaining term is the part attributable to imputation credits.

SFG then implicitly argue that the market risk premium in a non-imputation regime (denoted *MRP*) would be equal to the term { } in an imputation regime, i.e.,

$$MRP = \left\{ [M\hat{R}P + R_f] \left[\frac{1-\tau}{1-\tau(1-\gamma)} \right] - R_f \right\} \qquad (2)$$

It follows that

$$M\hat{R}P = [MRP + R_f] \left[\frac{1-\tau(1-\gamma)}{1-\tau} \right] - R_f \qquad (3)$$

and this corresponds to SFG's equation on their page 10.

SFG also argue that the estimate of 6% for the market risk premium that is invoked by the QCA draws upon data from non-imputation regimes, and is therefore an estimate for *MRP* rather than $M\hat{R}P$. So, invoking equation (3) with $R_f = .0584$, $\tau = .30$ and the QCA's estimate for γ of .50, the implied value for $M\hat{R}P$ is

$$M\hat{R}P = [.06 + .0584] \left[\frac{1-.30(1-.50)}{1-.30} \right] - .0584 = .0854 \qquad (4)$$

Acknowledging a range in values for *MRP* of .048 - .072, the resulting range in values for $M\hat{R}P$ is .071 to .100.

SFG also argue that, following equation (2), a post-imputation estimate for $M\hat{R}P$ of 6% along with an estimate for γ of .50 implies a post-imputation estimate for *MRP* of 3.91%, and this is inconsistent with the evidence supporting the estimate of 6% for *MRP*.

3. The Assessment

3.1 The Principal Error

The arguments presented in the previous section are subject to a fundamental error, as follows. In moving from equation (1) to (2), SFG implicitly assert that the introduction of imputation does not alter *MRP* but it raises $M\hat{R}P$ because the latter now contains an additional (non-zero) term. Thus, for example, if *MRP* is 6% prior to imputation, the introduction of imputation leaves this parameter at 6%. Also, with γ rising from zero to .50, equation (3) implies that $M\hat{R}P$ rises from 6% to 8.54% (the calculation appears in (4)). However this belief that *MRP* is invariant to the introduction of imputation is completely fallacious. Instead, *MRP* will decline by an amount corresponding to the benefit of imputation. Thus, it is $M\hat{R}P$ rather than *MRP* that remains unchanged. Accordingly, the claim that $M\hat{R}P$ should be between 7.1% and 10% is rejected.

The explanation is as follows. Purely to simplify the explanation, assume that imputation does not change the level of *cash* dividends on the market portfolio and these cash dividends are not otherwise expected to change over time from their current level of *DIV* per year. Also, suppose that the introduction of imputation raises the level of $D\hat{I}V$ (dividends inclusive of the imputation credits, to the extent of being usable) by 30%. Regardless of whether imputation operates or not, the value of the market portfolio (*V*) can be expressed as the present value of the cash dividends using the discount rate k_m (being the discount rate applicable to cash dividends and capital gains). With no expected growth in cash dividends, the simple perpetuity formula applies as follows.

$$V = \frac{DIV}{k_m}$$

The introduction of imputation raises *V* but has no impact on the cash dividends *DIV*. So, the discount rate k_m falls (in recognition of a reduction in the personal taxes associated with the cash dividends). Accordingly, *MRP* falls.

One can also express the value of the market portfolio (both before and after the introduction of imputation) as the present value of $D\hat{I}V$ (dividends inclusive of imputation credits, to the extent of being usable) using the discount rate $\hat{k}_m$. For the simple perpetuity case, the result is as follows.

$$V = \frac{D\hat{I}V}{\hat{k}_m}$$

Imputation raises $D\hat{I}V$ by 30%, and therefore raises *V* also by 30%. It follows that the discount rate $\hat{k}_m$ is unchanged. Accordingly, $M\hat{R}P$ is unchanged.

In summary, and contrary to SFG's claim, imputation leads to a reduction in *MRP* whilst leaving $M\hat{R}P$ unchanged.

3.2 The Secondary Error

A secondary error in SFG's analysis is to assert (without proof) that the proportion of $\hat{k}_m$ that is attributable to imputation credits (denoted *p*) is

$$p = \frac{\gamma\tau}{1-\tau(1-\gamma)} \tag{5}$$

Since γ is the product of the utilisation rate for imputation credits (*U*) and the proportion of company taxes (*TAX*) that are assigned as imputation credits (*IC*), it follows that

$$p = \frac{U\frac{IC}{TAX}\tau}{1-\tau\left[1-U\frac{IC}{TAX}\right]} \tag{6}$$

To examine whether (5) and therefore (6) is correct, it is sufficient to examine one special case: that in which U = 1 and IC = TAX, i.e., all company taxes paid are assigned as imputation credits and investors are able to fully utilise these credits in reducing their personal tax liabilities. Thus, as TAX becomes infinitesimally small (due perhaps to increasingly successful tax avoidance by firms), equation (6) and therefore also (5) implies that the proportion p is unaffected. However, as TAX goes to zero (and reaches zero in the limit), imputation credits must become less and less significant (and zero in the limit). Accordingly p must decline as TAX goes to zero. So, equation (6) and therefore (5) also must be wrong.

The actual proportion of $\hat{k}_m$ that is accounted for by imputation credits is as follows. Following Lally (2004, pp. 20-21)

$$\hat{k}_m = k_m + UD_m \frac{IC_m}{DIV_m} \tag{7}$$

where k_m is the expected market return defined in the conventional way to exclude imputation credits, D_m is the cash dividend yield on the market portfolio, and IC_m/DIV_m is the ratio of attached imputation credits to dividends paid in respect of the market portfolio. Lally (2004, p 52) gives estimates for the last two parameters of .032 and .19 respectively. Coupling this with an estimate for U and $\hat{k}_m$ then yields the proportion of the latter due to imputation credits. Of course, as TAX goes to zero, IC_m will go to zero, and therefore the proportion of $\hat{k}_m$ attributable to the credits goes to zero.

This point has implications for the post-imputation differential between $M\hat{R}P$ and MRP, with SFG alleging a differential of 3.91%. Deducting R_f from both sides of equation (7) yields

$$\hat{MRP} = MRP + UD_m \frac{IC_m}{DIV_m}$$

Using the estimates noted above of .032 and .19 for the last two parameters, along with an estimate for *U* of .625 (this is implicit in the QCA's estimate of .50 for γ, as discussed in Lally, 2004, p 121), the result is as follows.

$$\hat{MRP} = MRP + .625(.032)(.19) = MRP + .0038$$

So, a post-imputation estimate for $\hat{MRP}$ of 6% implies an estimate for *MRP* of 5.62% rather than the 3.91% claimed by SFG. It is also worth noting that, if *MRP* were 6% prior to the introduction of imputation, then the introduction of imputation would have lowered it to 5.62%.

3.3 Empirical Implications

Having argued in section 3.1 that the introduction of imputation would lower *MRP*, and assessed the size of the reduction in section 3.2, it might be thought that these assertions could be tested by comparing data from the pre-imputation period with that since imputation. However, the size of the reduction in *MRP* induced by imputation is so small relative to the inherent variability in market returns that such a reduction could not be statistically identified, i.e., even if the average market returns net of R_f in the two periods exactly matched the values for *MRP* in the two periods, a statistical test would conclude that the difference was statistically insignificant.

To demonstrate this, define R_1 as the average annual market return (net of R_f) in the pre-imputation period and R_2 as that in the post-imputation period. Thus, a test of whether R_1 is statistically significantly different from R_2 is equivalent to a test of whether $R_1 - R_2$ is statistically significantly different from zero. Using the *t*-test, along with the expected difference in average annual returns of .38%, the following *t* statistic arises

$$t = \frac{.0038}{\sqrt{Var(R_1 - R_2)}}$$

where $Var(R_1 - R_2)$ is the variance of $R_1 - R_2$. This t statistic would need to exceed 2 for a statistically significant result to arise. Since annual returns are independent it follows that

$$Var(R_1 - R_2) = Var(R_1) + Var(R_2) = \frac{\sigma^2}{N_1} + \frac{\sigma^2}{N_2}$$

where σ is the standard deviation of annual market returns net of the risk free rate, N_1 is the number of years of data in the pre-imputation period and N_2 the number in the post-imputation period. The longest such periods for which data are available are 1900-1987 and 1988-2004, and these will be assumed, i.e., $N_1 = 87$ and $N_2 = 16$. Also, the standard deviation of annual market returns is about .20 (Lally, 2004, p 53)[1]. So, substitution into the last two equations yields

$$Var(R_1 - R_2) = \frac{.20^2}{87} + \frac{.20^2}{16} = .003$$

and therefore

$$t = \frac{.0038}{\sqrt{.003}} = .07$$

The t-statistic is then far less than the value of 2 required to generate a statistically significant result. Even if there were 87 years of data post-imputation as well as pre-imputation, the t statistic would rise to only .125. In fact, to obtain a t statistic as large as 2, one would require 22,000 years of data both pre and post imputation! Thus, it is simply impossible to identify a differential of .38% in average annual returns in the pre and post-imputation periods as statistically significant.

4. Implications of Consistency

SFG implicitly argues that any estimate for $M\hat{R}P$ must be consistent with that for γ. I agree with this point, although the relevant parameter is the utilisation rate U rather than γ. The QCA considers that an appropriate estimate for $M\hat{R}P$ is 6%, and cites Lally (2004, section 6) in support of this. However the QCA also favours an estimate

[1] Subtraction of R_f has no material impact upon this standard deviation, due to the low variability in R_f.

for U of .625, and this contrasts with the estimate for U of 1 that underlies Lally's estimate of 6% for $M\hat{R}P$. This gives rise to a suggestion of inconsistency on the part of the QCA, the effect of which can be resolved by ascertaining whether Lally's estimate of $M\hat{R}P$ would have differed if he had invoked an estimate for U of .625 rather than 1.

Lally's estimate for $M\hat{R}P$ arises from considering a number of estimation methods. The first of these is historical averaging of the Ibbotson type (Lally, 2004, section 6.1). This involves averaging over the annual results for

$$R_m + UD_m \frac{IC_m}{DIV_m} - R_f \tag{8}$$

where R_m is the market return exclusive of imputation credits. As noted by Lally (ibid, pp. 51-52):

"Applying the Ibbotson methodology, with arithmetic averaging and long-term bond yields (10 yr), Dimson et al (2002, Table 18-1) estimates the Australian market risk premium at .075, using data from 1900-2000. This data omits inclusion of the central term in equation (8). However, since this term applies only since 1987, the omission exerts only a minor effect on the average across the full 100 years of data. To see this, the current value for the central term in equation (8) involves a value for U of 1, a market dividend yield of .032, and a franking rate of .19. The product is .006. If it is attributed to each of the 13 years since the introduction of imputation, the effect upon the estimate of (8) is to raise it by less than .001. In addition to this data issue, the introduction of imputation in 1987 would have introduced a regime shift (downwards) in k_m. However, as noted by Officer (1994, p. 10) this should be equal to the central term in equation (8) so that (8) would have been invariant to the regime shift."

If a value for U of .625 substitutes for a value of 1, the conclusion here is unaffected, i.e., the upward adjustment to the Dimson et al figure of .075 is still less than .001 and therefore the Dimson et al figure is still appropriate.

The second estimation method considered by Lally was that of the Siegel type, which involves replacing the average real bond yield embedded in the Ibbotson type estimate with an estimate of the long-run expected real bond yield (Lally, 2004, section 6.1). The latter two figures are .019 and .040 respectively. Combining these with the Ibbotson type estimate of .075 yields the following estimate of $M\hat{R}P$:

$$M\hat{R}P = .075 + .019 - .040 = .054$$

The only aspect of this estimate affected by the estimate for *U* is the Ibbotson-type estimate of .075. As discussed in the previous paragraph, reducing the estimate for *U* from 1 to .625 has no material effect on it. So, the same conclusion applies to the Siegel type estimate of .054.

The third estimation method considered by Lally is that in which the market risk premium is considered to be proportional to market volatility. Using estimates for the Australian market variance of $.183^2$ and the coefficient applied to this of 2, the resulting estimate of the market risk premium is

$$2\left(.183^2\right) = .067$$

To this estimate, Lally adds the following comments (ibid, pp. 58-59):

"This is an estimate of the standard market risk premium, i.e., $k_m - R_f$. If the data used to estimate the reward to risk ratio (estimated at 2) were drawn from the Australian market in the period since imputation was introduced, the estimate of .067 would require addition of the central term in equation (8). This would raise the .067 figure by about .006. If the data were drawn from the Australian market prior to the introduction of imputation, no adjustment would be required because the standard premium in the pre-imputation period should be equal to the Officer premium in the post-imputation period. However the data are drawn from a variety of markets, some with imputation and some without. Even in markets with imputation (such as Australia) the data is drawn largely from the pre-imputation period. Thus the figure of

.067 requires some adjustment, but by much less than .006. This suggests an estimate for the market risk premium of about .07. In addition, the estimates for the risk-to-reward ratio involve the use of short-term risk free rates, and this is not consistent with the use of the ten year risk free rate in other estimates presented here. Since the estimated reward-to-risk ratio of 2 is drawn from a number of markets, the appropriate adjustment to the figure of .07 is unclear, but is likely to be downwards in view of the fact that longer term interest rates are generally higher than shorter term rates."

Thus, if the estimate for U were lowered from 1 to .625, the effect would be to lower the estimate of the market risk premium in the Officer model, but the degree would be as slight as in the earlier Ibbotson and Siegel type estimates.

The last estimation method considered by Lally was the forward-looking methodology of Cornell (1999). This involves estimating k_m so as to equate the present value of future cash dividends on the market portfolio with their current market value, followed by substitution into the formula for the market risk premium, i.e.,

$$M\hat{R}P = k_m + UD_m \frac{IC_m}{DIV_m} - R_f \qquad (9)$$

Lally (2004, p 62) presents a range of estimates for k_m, along with values for the remaining four parameters of 1, .032, .19 and .062. The range in estimates for k_m gives rise to a range of estimates for $M\hat{R}P$ from .040 to .057. So, regardless of the estimate for k_m, the effect of lowering the estimate for U from 1 to .625 will be to lower the central term in (9) by .002, and therefore the estimate of $M\hat{R}P$ by the same amount[2].

In summary, and considering the four estimation methods invoked by Lally, the effect upon the estimate of $M\hat{R}P$ from lowering the estimate for U from 1 to .625 is immaterial in three of those cases and downwards by .002 in the fourth case. In my view this is insufficient to warrant any shift in the estimate for $M\hat{R}P$ of 6%, but any

[2] Using an estimate for U of 1, the central term in (9) has a value of .006. Reducing the estimate of U to .625 reduces the central term in (9) to .004.

shift would be downwards rather than upwards. So, even if there is an inconsistency between the QCA's estimate of 6% for $M\hat{R}P$ and .625 for *U*, the required adjustment to $M\hat{R}P$ would be both immaterial and downwards rather than upwards.

5. Conclusion

This paper has reviewed arguments raised by SFG (2004), and the conclusions are as follows. Firstly, SFG's principal point assumes that the introduction of imputation leaves *MRP* unaffected and raises $M\hat{R}P$, and this is completely incorrect. In fact, the introduction of imputation would reduce *MRP* by the level of imputation credits and therefore leave $M\hat{R}P$ unaffected. Secondly, SFG's formula for determining the post-imputation difference between *MRP* and $M\hat{R}P$ significantly overestimates the difference. Rather than the 2.1% differential suggested by SFG, the differential is less than .40%. Thirdly, SFG's argument that the estimate for *U* should be consistent with that for $M\hat{R}P$ is correct, and it could be argued that there is some inconsistency in the QCA's estimates of .625 and 6% respectively. However, any adjustment to the latter would be immaterial and downwards rather than upwards.

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